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REVIEW PAPER

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STRENGTH AND DILATANCY OF SANDS IN DIRECT SHEAR

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ABSTRACT

This article analyses the strength and dilatancy of three sands with different morphologies using four established stress–dilatancy theories. The morphology of granular materials, including sands, significantly influences their physical, mechanical, and hydraulic behaviour, mainly due to the influence of grain shape. Sands with more angular grains demonstrated higher strength and dilatancy than the ones with rounded grains. Analysing the influence of relative density and stress levels on the strength and dilatancy of sands in direct shear tests indicates that an increase in relative density enhances sand strength and dilatancy, while an increase in stress levels reduces these properties. The essential influence of grain morphology, relative density, and stress level on sand strength provides valuable insights for geotechnical applications concerning granular materials.

Keywords: sands, strength, dilatancy, direct shear

INTRODUCTION

It is important to understand the behaviour of sands under shear stress due to its direct implication on the stability of civil and geotechnical engineering structures such as slopes, embankments, retaining walls, and foundations. Granular materials such as sands tend to undergo volume changes during shearing, and this is called dilatancy (Dołżyk-Szypcio, Szypcio, Godlewski & Witowski, 2024). The complex relationship between strength and dilatancy is pivotal to understanding the factors affecting the behaviour of sands under shearing and can be described using the stress–dilatancy plane (Szypcio, 2017b; Szypcio, Dołżyk-Szypcio & Nurgaliyev, 2022a). The residual friction angle (ϕ_r) is used to describe the strength of sand under conditions of zero dilatancy, where the soil exhibits no change in volume during shearing, representing the residual shear strength in direct shear (Houlsby, 1991). Meanwhile, the factors that may affect the shearing resistance may include, but are not limited to, the inter-particle friction, distribution of particle size, particle shape, composition, surface texture, and density (Peng, Chen & Wu, 2021; Wang et al., 2021; Anusree & Latha, 2023).

The macroscopic properties of sands have been studied theoretically and in the laboratory by various researchers (Rowe, 1962; Lee & Seed, 1967; De Josselin De Jong, 1976; Scarpelli & Wood, 1982). Numerical discrete element modelling (DEM) has also been employed to study the micro-mechanical interactions and properties of sand (Wang, Dove & Gutierrez, 2007; Zhang & Thornton, 2007; Gutierrez & Wang, 2010; Amirpour Harehdasht, Hussien, Karray, Roubtsova & Chekired, 2019). It can be observed that the maximum shear strength of sand is notably influenced by the stress level and initial relative density (Dołżyk, 2006),

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and the stress–dilatancy relationship, as well as stress–strain behaviour is dependent on moisture content, normal stress, stress level and degree of compaction (Szypcio, 2017b; Dołżyk-Szypcio, 2019a). The most common theories of dilatancy proposed by Taylor (1948), Rowe (1962), Bolton (1986), Houlsby (1991), and Szypcio (2017) can correctly describe the relationship between the stress ratio and dilatancy (Dołżyk-Szypcio, 2020).

In this paper, the results of direct shear tests of three sands of different origins and grain properties published by Guida, Sebastiani, Casini and Miliziano (2019) will be analysed. The critical state frictional angle (ϕ^o) is independent of the stress level (Szypcio, 2023), but the influence of different stress levels and relative density on the strength of the sand will be analysed using four main stress–dilatancy theories (Taylor, 1948; Bolton, 1986; Houlsby, 1991; Szypcio, 2017b).

STRENGTH–DILATANCY IN DIRECT SHEAR

Following the pioneering work by Casagrande (1936), Taylor (1948) proposed a relationship between frictional angle and dilatancy. He suggested that all frictional relationships result from the internal dissipation of energy and that the dissipated energy is directly proportional to shear strain rate and normal stress. This relationship is known as the flow rule:

$$\delta W = \sigma'_n \delta\varepsilon_v + \tau\delta\gamma = (\tan\phi_r)\sigma'_n \delta\gamma \tag{1}$$

or

$$\frac{\tau}{\sigma'_n} = \left(\frac{\tau}{\sigma'_n}\right)_r + \left(\frac{\delta h}{\delta s}\right) \tag{2}$$

or

$$\tan\phi_{\text{peak}} = \tan\phi_r + \tan\psi, \tag{3}$$

where:
σ'_n – effective normal stress [kPa],
$\delta\varepsilon_v$ – volumetric strain increment [-],
τ – shear stress [kPa],
$\delta\gamma$ – shear strain increment [-],
ϕ_r – residual friction angle [°],
δh – sample height increment [mm],
δs – shear displacement increment [mm].

$$\tan\psi = \frac{-\delta\varepsilon_v}{\delta\gamma} = \frac{\delta h}{\delta s}, \tag{4}$$

where:
ψ – dilatancy angle [°].

$$\frac{\tau}{\sigma'_n} = \tan\phi_{\text{peak}}, \tag{5}$$

where:
ϕ_{peak} – peak frictional angle [°].

After analysing a series of experiments presented in the literature, Bolton (1986) proposed an empirical approach to the relationship between frictional angle and dilatancy. This empirical flow rule correlation suggests that any shear strength above the residual shear angle at a critical state is due solely to dilatancy, which occurs as a result of volume expansion and particle rearrangement:

$$\phi_{\text{peak}} = \phi_{cv} + 0.8\psi, \tag{6}$$

where:
ϕ_{cv} – critical state angle [°].

The sawtooth model proposed by Houslby (1991) considered frictional blocks sliding over another on a rough plane (sawtooth) at a dilatancy angle (ψ) to the horizontal and residual frictional angle (ϕ_r) – at constant volume or critical state – acting on the saw teeth. The relationship between shear stress and normal stress can be expressed as:

$$\frac{\tau}{\sigma_n'} = \tan\phi_{\text{peak}} = \tan\left(\phi_r + \psi\right) \tag{7}$$

or

$$\phi_{\text{peak}} = \phi_r + \psi. \tag{8}$$

Szypcio (2017b) proposed a stress–dilatancy relationship based on the frictional state concept. Equation 9 from Szypcio (2017b) correctly describes the stress–dilatancy relationship at failure in the general state (Szypcio, Dołżyk-Szypcio & Mierczyński, 2022b):

$$\tan\phi_{\text{peak}} = \frac{\tau}{\sigma_n'} = \frac{\sqrt{3}\,\eta\cos\phi^o\cos\theta}{3+\eta\,(\sin\theta - \sqrt{3}\sin\phi^o\cos\theta)}, \tag{9}$$

where:

$$\eta = \mathrm{M}_b^o - \mathrm{A}_b^o\left(\alpha + \beta D^p\right), \tag{10}$$

α, β – parameters of the frictional state concept representing the mode of deformation ($\alpha = 0$, $\beta = 1.4$),
θ – Lode angle for stress ($\theta = 15°$).

Lode angle and parameters α and β are for granular soils under plane strain (direct shear) conditions (Szypcio, 2017b).

For sands under plane strain conditions, parameters could be calculated as follows:

$$\phi^o = \phi_{cs} = \phi_r, \tag{11}$$

$$M_b^o = \frac{(3\sin\phi^o)}{\sqrt{3}\cos\theta} - \sin\phi^o\sin\theta. \tag{12}$$

For sands in drained conditions, parameters could be calculated as follows:

$$\mathrm{A}_b^o = \frac{1}{\cos\left(\theta - \theta_\varepsilon\right)}\left\{1 - \frac{2}{3}\mathrm{M}_b^o\sin\left(\theta + \frac{2}{3}\pi\right)\right\}, \tag{13}$$

$$\tan\theta_\varepsilon = \frac{1}{\sqrt{3}}\frac{\left(\frac{\delta h}{\delta s}\right)}{\sqrt{1+\left(\frac{\delta h}{\delta s}\right)^2}}, \tag{14}$$

$$D^p = \frac{\delta\varepsilon_v}{\delta\varepsilon_q} = -\sqrt{3}\frac{\left(\frac{\delta h}{\delta s}\right)}{\sqrt{1+\frac{4}{3}\left(\frac{\delta h}{\delta s}\right)^2}}, \tag{15}$$

where:
η – stress ratio [-],
$\delta\varepsilon_v$ – volumetric strain increments [-],
$\delta\varepsilon_q$ – shear strain increments [-],
D^p – plastic dilatancy [-].

CHARACTERISTICS OF ANALYSED SANDS

Guida et al. (2019) conducted a series of direct shear tests on sands of different origins and properties to study the relationship between micro-grain shape features and macro-mechanical behaviour of the sands under shearing. The sands have the same mono-disperse grain-size distribution with a grain diameter ranging between 1.0 to 1.6 mm and were prepared by dry pluviation and mechanical vibration. They were sheared in a direct shear box apparatus at a constant displacement rate of 0.18 mm·min^{-1} under three vertical stresses (σ_n equal 50, 100 and 200 kPa). Three of the sands will be analysed in this study and are selected on the basis of their morphology and geotechnical properties in order to understand the relationship between their grain shape, strength, and dilatancy. Bucarest, Fumone, and Torre del Lago sands were selected for this study. Bucarest sands are natural sands obtained from the subsurface, while Fumone and Torre del Lago sands are artificially ground and quarry-sourced, respectively.

The morphology of the sands varied significantly and reflects the geotechnical properties, shear strength, and dilatancy. Fumone sands have very angular and low sphericity grains with the highest void ratio of the three due to their irregular grains (e_{min} = 0.87, e_{max} = 1.31) and a critical state angle (ϕ_{cs}) of 37.0°. On the other hand, Torre del Lago sands comprise rounded and high sphericity grains with a void ratio of e_{min} = 0.64, e_{max} = 0.92 and the lowest shear strength with a critical state angle (ϕ_{cs}) of 26.5°. Meanwhile, Bucarest sands have sub-angular grains with high sphericity as well as a void ratio similar to Torre del Lago sands (e_{min} = 0.65, e_{max} = 0.92). In contrast, its shear strength and critical state angle (ϕ_{cs} = 30.9°) are higher than Torre del Lago sands but lower than Fumone sands. The main properties of the analysed sands are summarised in Table 1. The parameters A and α in Table 1 were calibrated against the experimental data.

Table 1. Morphological and mechanical properties of the sands

Sand name	Morphology	G_s [kg·m^{-3}]	e_{min} [-]	e_{max} [-]	ϕ_{cs} [°]	A [-]	α [-]
Bucarest	sub-angular, high sphericity	2.70	0.65	0.92	30.9	12.95	0.25
Fumone	very angular, low sphericity	2.91	0.87	1.31	37.0	15.17	0.82
Torre del Lago	rounded, high sphericity	2.70	0.64	0.92	26.7	0.31	1.01

G_s – specific gravity, e_{min} – void ratio of the soil in the densest state, e_{max} – void ratio of the soil in the loosest state, ϕ_{cs} – critical state angle, A, α – experimental parameters.

Source: Guida et al. (2019).

From Guida et al. (2019) in direct shear:

$$\psi = A D_r \left(\frac{\sigma_n}{p_a} \right)^{-\alpha}, \tag{16}$$

where:
ψ – dilatancy angle [°],
A, α – experimental parameters [-].
D_r – relative density [%],
σ_n – normal stress [kPa],
p_a – atmospheric pressure (p_a = 101) [kPa],

The dilatancy angle (ψ) is a function of the granular materials' properties (A, α), relative density (D_r), and stress level (σ_n/p_a).

INFLUENCE OF RELATIVE DENSITY ON SAND STRENGTH

The influence of relative density on the strength of Bucarest, Fumone, and Torre del Lago sands was analysed under different normal stresses (σ_n). Figures 1–3 show the relationship between ($\phi_{\text{peak}} - \phi_r$) − D_r. The results of the influence of relative density on the sand strength presented in these graphs was calculated using the equations of different stress–dilatancy theories described earlier.

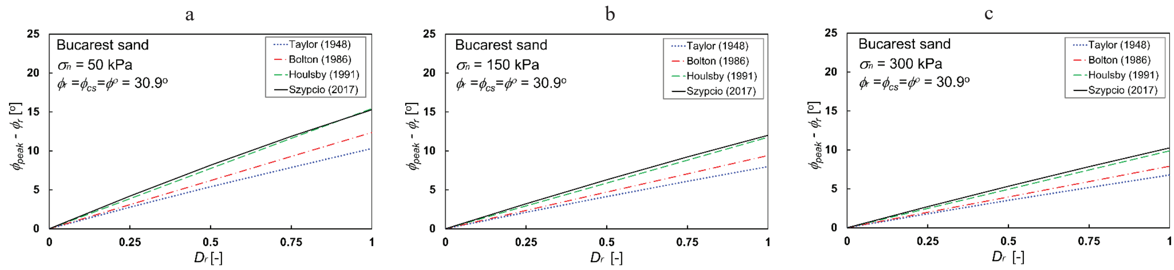

Fig. 1. The influence of relative density (D_r) on Bucarest sand strength ($\phi_{\text{peak}} - \phi_r$) for: a − σ_n = 50 kPa, b − σ_n = 150 kPa, c − σ_n = 300 kPa

Source: author's work.

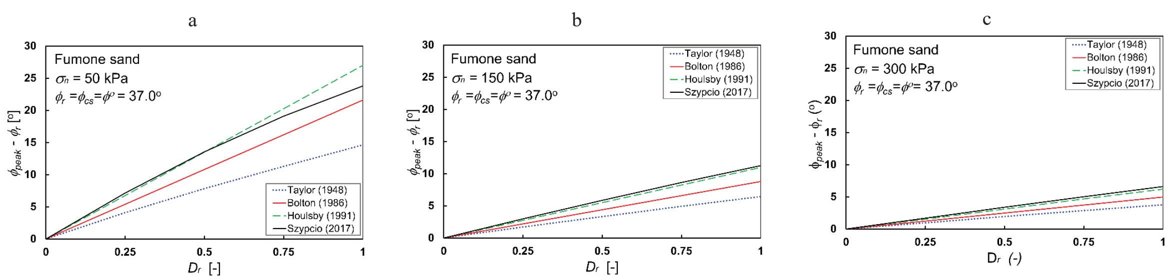

Fig. 2. The influence of relative density (D_r) on Fumone sand strength ($\phi_{\text{peak}} - \phi_r$) for: a − σ_n = 50 kPa, b − σ_n = 150 kPa, c − σ_n = 300 kPa

Source: author's work.

For Bucarest sand, an increase in relative density from 0 to 1.0 shows a progressive rise in the difference between peak and residual friction angle ($\phi_{\text{peak}} - \phi_r$). Using the Szypcio (2017b) model for σ_n = 50 kPa, $\phi_{\text{peak}} - \phi_r$ reaches approximately 15° at D_r = 1.0, compared to 7° at D_r = 0.25. A similar trend is observed for the other theories, and at higher stress levels, σ_n equals 150 kPa and 300 kPa. However, the Taylor (1948) model is least responsive to relative density, rising only to 10° at D_r = 1.0.

Fumone sands exhibit the most pronounced effect of relative density on sand strength (ϕ_{peak}) due to the angularity of its grains. At σ_n = 50 kPa, the Houlsby model showed that ($\phi_{\text{peak}} - \phi_r$ increases from 0 to 27° when the relative density (D_r) is increased from 0 to 1. Szypcio's (2017b) model also showed a similar increase with $\phi_{\text{peak}} - \phi_r$ rising to 25° when the relative density is 1. The Bolton (1986) model reaches 21° while the Taylor (1948) model remains the least sensitive, reaching 14.6°.

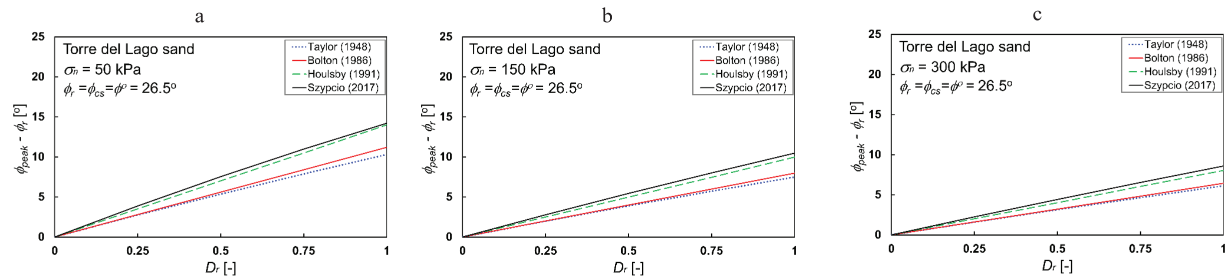

Fig. 3. The influence of relative density (D_r) on Torre del Lago sand strength ($\phi_{\text{peak}} - \phi_r$) for: a – σ_n = 50 kPa, b – σ_n = 150 kPa, c – σ_n = 300 kPa

Source: author's work.

Unlike Bucarest and Fumone, Torre del Lago sands show the least impact of relative density on sand strength due to its rounded grains that cause weaker interlocking effects and less resistance to deformation, which makes dilatancy less influential at all stress levels.

For all analysed tests, the effect of relative density on sand strength is evident with an increase in the difference between peak frictional angle and critical frictional state angle ($\phi_{\text{peak}} - \phi_r$), although the magnitude of this effect varies depending on the sand type and theoretical model. This denotes an increased sand strength and dilatancy effect in denser sands.

Taylor's (1948) model is less sensitive to relative density compared to other theories, suggesting that it may underestimate the influence of particle rearrangement and interlocking. However, Bolton (1986) shows a better effect of relative density on sand strength, especially in moderately dense sands (D_r = 0.5, D_r = 0.75). In very dense sands (D_r = 1.0), more detailed micromechanical theories like Houlsby (1991) and Szypcio (2017b) show a better influence on sand strength. Houlsby (1991) shows a strong influence of relative density on sand strength as it accounts for particle interlocking and geometric resistance during shearing. This is evident in angular sands like Fumone, where interlocking plays a key role in dilatancy. Frictional state theory by Szypcio (2017b), which emphasises the relationship between dilatancy and critical state theory, effectively shows the influence of relative density on sand strength. The more angular sands (Fumone) show higher sand strength, especially at σ_n = 50 kPa with $\phi_{\text{peak}} - \phi_r$ = 27°; meanwhile, Bucarest and Torre del Lago sands exhibit moderate sand strength with increased relative density.

INFLUENCE OF STRESS LEVEL ON SAND STRENGTH

The effects of stress levels on the strength and dilatancy of Bucarest, Fumone, and Torre del Lago sands were analysed across various relative densities. In Figures 4–6, the relationship between the stress level and sand strength is shown for D_r of 0.25, 0.5, 0.75, and 1.0 using the four stress–dilatancy theories already discussed.

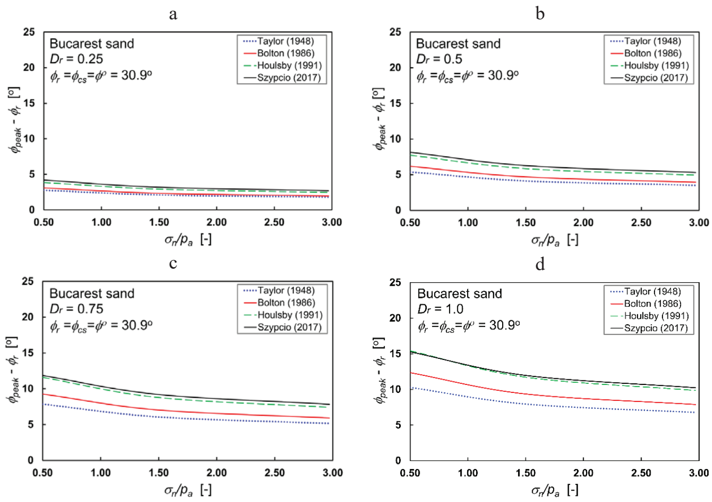

Fig. 4. The influence of stress level (σ_n/p_a) on Bucarest sand strength ($\phi_{\text{peak}} - \phi_r$) for: a − D_r = 0.25, b − D_r = 0.5, c − D_r = 0.75, d − D_r = 1.0

Source: author's work.

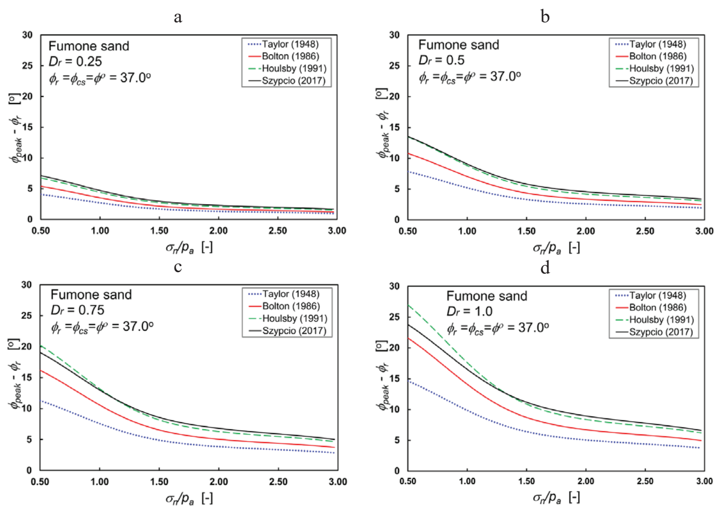

Fig. 5. The influence of stress level (σ_n/p_a) on Fumone sand strength ($\phi_{\text{peak}} - \phi_r$) for: a − D_r = 0.25, b − D_r = 0.5, c – D_r = 0.75, d − D_r = 1.0

Source: author's work.

The effect of stress level on Bucarest sand shows a gradual reduction in sand strength across all models. The Houlsby (1991) and Szypcio (2017b) models capture this decline effectively with $\phi_{peak} - \phi_r$ reducing from approximately 15° at stress level (σ_n/p_a) = 0.5°–10° at σ_n/p_a = 2.97. Taylor (1948) and Bolton (1986) also show this decline.

The effects of stress levels on Fumone sands are clearly evident, indicating that angular grains lose much of their interlocking at high normal stress. It can be observed that at lower stress levels, Fumone sands have the highest sand strength, but with an increase in stress level, there is a steeper reduction in sand strength. At D_r = 1, $\phi_{peak} - \phi_r$ reduced from 27° at σ_n/p_a = 0.5 to approximately 6° at σ_n/p_a = 2.97 using Szypcio's (2017b) model. A similar steep reduction can be observed across all models and relative densities.

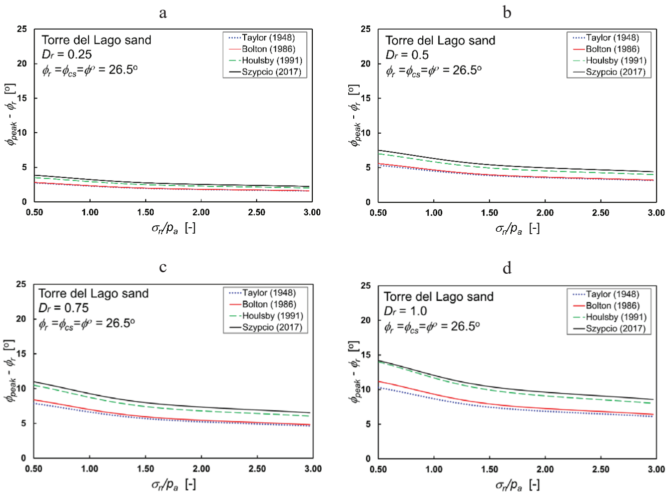

Fig. 6. The influence of stress level (σ_n/p_a) on Torre del Lago sand strength ($\phi_{peak} - \phi_r$) for: a − D_r = 0.25, b − D_r = 0.5, c − D_r = 0.75, d − D_r = 1.0

Source: author's work.

Among all the sands, Torre del Lago suffered the least noticeable reduction in sand strength when the stress level was increased. This can be attributed to its rounded grains, as an increase in stress level has only considerable effects on its interparticle friction due to reduced contributions of dilatancy. At D_r = 1, $\phi_{peak} - \phi_r$ reduces from 14.2° at σ_n/p_a = 0.5 to only about 8.5° at σ_n/p_a = 2.97, signifying a linear reduction in the strength of sand. This pattern is consistent across all densities and models.

In Figures 4–6, the influence of stress level on sand strength is characterised by a decrease in $\phi_{peak} - \phi_r$ as the stress level is increased. This denotes the reduction in sand strength and dilatancy under high normal stresses, where grain crushing may occur and particle rearrangement may be limited.

Fumone sands show the highest strength but also the steepest reduction as stress levels increase. This is due to the angular nature of the sands that makes them dilatant at low stress but become susceptible to particle breakage at higher confining stress levels, as evidently shown in Houlsby (1991) and Szypcio (2017b). Bucarest and Torre del Lago sands demonstrate lower influence on stress levels due to more rounded grains, with the latter demonstrating the least. It can also be seen from Figures 4–6 that the differences among the results from the theories of different researchers are relatively small.

Taylor (1948) underestimates the rate of strength reduction with increasing stress levels, especially in angular grain sands such as Fumone. However, Bolton (1986) captures stress dependence more effectively, showing a steeper decline in strength than the Taylor (1948) model. Szypcio's (2017b) frictional state concept and the sawtooth model by Houlsby (1991) provide a more nuanced representation, exhibiting the steepest decline in strength with an increase in stress level. Houlsby (1991) showed a sharp decrease in $\phi_{peak} - \phi_r$ at lower stress levels (σ_n/p_a = 0.5–1.5) and gradually flattens out as the stress level increases (σ_n/p_a = 1.5–3.0), indicating a reduction in dilatancy at higher confining stress levels. This model is particularly pronounced in sands with angular grains like Fumone, where grain morphology greatly affects strength. Similarly, Szypcio (2017b) shows the steepest reduction in strength with stress, reflecting a transition into critical state conditions at high stress levels where dilatancy effects are minimised and the sand approaches its residual state.

CONCLUSIONS

This study examined the influence of relative density and stress level on the strength of Bucarest, Fumone, and Torre del Lago sands. The findings revealed that the strength of sands is significantly influenced and enhanced by an increase in relative density, with a noticeable increase in the difference between $\phi_{peak} - \phi_r$ as D_r increases from 0 to 1.0. This increase was more pronounced in very angular sands like Fumone, where its morphology contributed to an increase of up to 27° at D_r = 1.0 under low stress conditions (σ_n = 50 kPa), compared to Bucarest and Torre del Lago, which exhibited increases of around 16° and 14°, respectively. Houlsby's (1991) and Szypcio's (2017b) models consistently predicted higher strength gains, emphasising the significant role of dilatancy in dense sands.

Conversely, the increase in stress level (σ_n/p_a) reduces the sand strength, particularly under high stress levels where dilatancy reduces and particle breakage occurs. It was observed that $\phi_{peak} - \phi_r$ decreased with a rise in σ_n for all sands; this reduction was steepest for very angular sands such as Fumone, where at D_r = 0.75, $\phi_{peak} - \phi_r$ reduced from 20° at 50 kPa to 5° at 300 kPa, as captured by the Houlsby (1991) and Szypcio (2017b) models. In contrast, Torre del Lago sand shows less pronounced reduction due to its rounded grains, and Bucarest sand shows moderate reductions.

For all sands, Taylor (1948) and Bolton (1986) tend to underestimate the strength changes as density and stress levels are changed. In comparison, Houlsby (1991) and Szypcio (2017b) provide better predictions of the sand strength, particularly for dense sands under varying stress conditions. Angular sands like Fumone exhibit the highest sensitivity to both density and stress level, while rounded sands like Torre del Lago show the least sensitivity and Bucarest sands display moderate strength characteristics.

Overall, the results of this study showed the complex relationship between relative density, stress level and grain morphology in determining the strength of sands. These insights are important in modelling sand behaviour in engineering and optimising designs under varying density and stress conditions.

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WYTRZYMAŁOŚĆ I DYLATANCJA PIASKÓW W BEZPOŚREDNIM ŚCINANIU

STRESZCZENIE

W artykule analizowano wytrzymałość i dylatancję trzech piasków o różnej morfologii w świetle czterech wybranych teorii opisujących współzależność naprężenia i dylatancji. Morfologia materiałów ziarnistych, w tym piasków, znacząco wpływa na ich cechy fizyczne, mechaniczne i hydrauliczne, głównie ze względu na wpływ kształtu ziaren. Piaski o ziarnach bardziej ostrokrawędzistych wykazały większą wytrzymałość i dylatancję niż te o ziarnach zaokrąglonych. Analiza wpływu gęstości względnej oraz poziomów naprężeń na wytrzymałość i dylatancję piasków w badaniach bezpośredniego ścinania wskazuje, że wzrost gęstości powoduje zwiększenie wytrzymałości i dylatancji piasku, podczas gdy wzrost poziomu naprężeń powoduje ich zmniejszenie. Wpływ morfologii ziaren, gęstości względnej i poziomu naprężeń na wytrzymałość piasku dostarcza cennych spostrzeżeń dotyczących materiałów ziarnistych.

Słowa kluczowe: piaski, wytrzymałość, dylatancja, bezpośrednie ścinanie