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SIZING OPTIMISATION OF STEEL TRUSS BASED ON ALGORITHMS

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ABSTRACT

Computational methods enable mathematical discretisation in structural design. Thus, thanks to the algorithmic design, the obtained results matrix presents various structurally efficient load-bearing elements. The main aim of this paper is to address the topic of material optimisation in truss bar structures with different approaches. The paper analyses and compares steel truss optimisation based on two software optimisation algorithms (MS Excel and Ansys Mechanical) using gradient and sub-problem methods. The key findings present how effective each method is in structural optimisation and concludes the present study with a roadmap to efficient structural designing of the load-bearing truss elements.

Keywords: structural optimisation, computer methods in structural engineering, truss optimisation, finite elements method

INTRODUCTION

Computer programs make it possible to consciously apply selected methods that, by understanding the working principles of engineering structures, give engineers the expected results and enable creative professional activity. A structure is a real object with inherent errors in the components' manufacturing and assembling process and has unavoidable imperfections. The basic model of the structure is created using theoretical assumptions and simplifications that result from theories describing physical phenomena so that the object's basic scheme of operation can be adopted, for example: frames, plates, and shells. The computer model is formed by approximate discretisation, which is necessary since computers operate with numbers, not functions (Logg, 2007). The essence of modelling is a simplification in which we analyse the dominant effects and omit the less important ones. Without losing the accuracy of the solution, it is usually possible to reduce the spatial structure and complex stress state to a one- or two-dimensional system. Subsequent models are generally more far-reaching simplifications created for specific theories or methods. It is worth noting that even the simplest structural systems would be impossible to solve without simplification.

Computer methods include the transition method from a continuum to a discrete system and the solution algorithm. Their common feature is the reduction of differential equations (e.g., equilibrium equations) to corresponding systems of algebraic equations. On the other hand, the main differences in computer methods concern the manner of discretisation and approximation of the function sought (Logg, 2007).

Even before the advent of computers, approximate methods for solving differential equations were developed and are still used today. They use mathematical discretisation because the coefficients have

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no direct counterparts in physical/mechanical quantities. Examples include the development of functions into the Taylor series, Fourier series, Ritz and Galerkin approximation methods and the finite difference method (approximation of derivatives by differential quotients). When creating the finite element analysis (FEA) model, discretisation involves dividing an area into elements that share nodes and contain mutual neighbourhood information. On the continuum, one looks for a function $f(x, y)$ that satisfies certain differential equations. In the discrete model, on the other hand, the search is for a set (vector) of numbers $f(x_i, y_i)$ at selected points on the continuum, which satisfies a mostly linear system of equations.

Classical methods of structural analysis, such as the force, displacement, or cross methods, have also played a significant role in developing computational methods. They use physical discretisation (unknown degrees of freedom of the system correspond to actual physical quantities, such as generalised displacements of nodes). The most widely used and comprehensive method in computerised structural analysis today is the finite element method (FEM). Alternatives to FEM include the boundary element method, meshless/meshfree methods and mathematical discretisation methods (Kirby, Logg, Scott & Terrel, 2006).

STATE OF THE ART

Environmental pollution

Architectural objectives include designing features such as usability, aesthetics and environmental processes (Kurcjusz, Stefańska, Dixit & Starzyk, 2022). Steel manufacturing industries are responsible for consuming large amounts of energy and fossil fuels; they are also a significant part of greenhouse gas emissions (Burchart-Korol, 2013). It is considered that around half of the greenhouse gas emissions are related to structural building materials (Webster et al., 2012). In recent years, there have been attempts to use alternative sustainable materials that would not negatively impact the environment (Vijayan et al., 2022).

According to studies, the construction sector strongly impacts environmental changes. In the European Union, construction is responsible for about 50% of natural resource consumption, in addition to the total energy consumption of 42%, greenhouse gas emissions of 35%, and waste flows of 32% (Pomponi & Moncaster, 2016). A significant portion of global greenhouse gas emissions comes from the production and processing of building materials. Despite this, a lot of material is wasted in construction due to over--dimensioned load-bearing structures. While numerical optimisation tools have the potential to decrease the usage of structural materials, their use in daily design practice is hindered by the absence of tailored algorithms (Dillen, Lombaert & Schevenels, 2021). By-products of construction activities pollute the environment – it is worth noting the energy consumed and carbon dioxide emissions consumed when transporting heavy and bulky construction components (Łacek & Starzyk, 2022).

Truss optimisation

Minimising the use of materials is possible through the efficient use of natural resources, which is the responsibility of today's construction sector (Dixit & Stefańska, 2022). The general role of optimisation is to find the function's minimum or maximum with respect to the restrictions. Shape, size and topology structural optimisation can be distinguished (Stolpe, 2016). Additionally, integrated optimisation consists of previously mentioned processes (Liu & Xia, 2022). The search for effects conditioned by the rationalisation of technical solutions is a challenge in determining the optimal form, especially in applying algorithmic design tools (Stefańska & Rokicki, 2022).

Structural size optimisations of truss elements aim to minimise the structure's total weight where the structural elements' cross-sectional area is a dimensional parameter – the design variable (Renkavieski & Parpinelli, 2021). Constraints – maximum stresses or deflections – must be considered when optimising the structure's weight (Kaveh & Zaerreza, 2020; Stefańska et al., 2022). Optimisation can allow reducing the use of materials, thereby lowering construction costs (Renkavieski & Parpinelli, 2021).

With numerical optimisation tools, automating tedious parts of the design process can restore the balance between rational design and material efficiency (Dillen, Lombaert & Schevenels, 2021). Structural optimisation is essential to designing a lightweight and efficient bar system that can safely carry loads (Liu & Xia, 2022). Truss optimisation is an engineering problem that can be approached in many different ways (Table 1). Due to its peculiarity, nonlinearity and multidimensional search space, a metaheuristic algorithm can be used (Renkavieski & Parpinelli, 2021). We can distinguish between deterministic and stochastic approaches in finding the best solution to a problem (Wang et al., 2013).

Table 1. The classification of optimisation tasks

Static optimisation	Dynamic optimisation
without restrictions	with restrictions
linear programming task	nonlinear programming task
smooth	non-smooth
continual	integer
deterministic	stochastic
with a single target function	multi-criteria

Source: the authors' compilation.

Gradient-based optimisation

Gradient-based methods can be used to solve continuous optimisation problems as it requires the objective and constraint functions to be differentiable (Haftka & Gürdal, 2012). To enhance the efficiency of optimisation algorithms, such methods utilise sensitivities or gradients of objective and constraint functions for optimal performance. However, the applicability of gradient-based techniques is contingent on the differentiability of objective and constraint functions, thus rendering them capable of handling solely continuous design variables like shape variables (Dillen, Lombaert & Schevenels, 2021).

Sub-problem method

In this method, a modified objective function is constructed at the outset, which contains information about the objective function's value and the penalties for exceeding the constraints. Then, from a randomly selected set of admissible points, this function is interpolated with a polynomial of an appropriate degree and its minimum is determined, which is an algebraically simple task. This process is repeated in subsequent iterations for a narrowed search area around the best solution found in the previous iteration. The method is, therefore, relatively simple, as it does not require the calculation of gradients of the objective and constraint functions, which can be expensive when estimated numerically. Thus, despite its simplicity, it performs well in many optimisation tasks.

MATERIALS

Research methodology

The case study analyses the steel truss structures with a sizing optimisation approach of the selected bars for the truss. The objective is to make the structure as light as possible when the boundary conditions for deflections and stresses are met, so optimising the cross-sections of the truss bars reduces the steel required for the truss. Finding the smallest possible volume assumes the same steel class in all bars. The decision variables will be the cross-sectional areas in the truss bars, which will be divided into four groups – top chord bars, bottom chord bars, vertical bars, and diagonal bars.

The objective function is linear – it is the sum of rod volumes. The minimum of this function is sought, where the decision variables are the areas of the bar groups (Eq. 1).

$$\min V(A) = \sum_{i=1}^{N} A_i \cdot L_i, \tag{1}$$

where:
V – volume of the structure,
A_i – cross-section area of the given bar,
L_i – length of the given bar.

Decision variables must be bounded by a minimum value (Eq. 2).

$$A_i \geq A_0, \tag{2}$$

where:
A_i – cross-section area of the given bar,
A_0 – minimum constraint cross-section area of the bar.

Stress conditions must be met – they must fit within certain limits. The stresses are inversely proportional to the decision variable – the cross-sectional area of each group of bars. Therefore, the constraints are no longer linearly dependent.

$$\sigma_i \leq |\sigma_0|, \tag{3}$$

where:
σ_i – stresses in the given bar,
σ_0 – maximum stresses that are allowed in the bar.

$$\sigma_i = \frac{N_i}{A_i}, \tag{4}$$

where:
σ_i – stresses in the given bar,
N_i – forces acting on the cross-section area,
A_i – cross-section area.

Displacements were calculated from the Maxwell–Mohr formula (Eq. 5).

$$w = \int_V \frac{\sigma^0 \cdot \sigma^1}{E} \mathrm{d}V = \sum_{i=1}^{N} \frac{N_i^0 \cdot \bar{N}_i^1 \cdot L_i}{E \cdot A_i} \leq w_0, \tag{5}$$

where:
w – deflection,
V – volume of the structure,
σ^0 – stress,
σ^1 – stress due to unit load,
E – Young modulus,
N_i^0 – internal force,
$\bar{N}_i^1$ – internal force due to unit load,
L_i – length of the given bar,
A_i – cross-section area,
w_0 – maximum constraint deflection.

The maximum deflection for a lattice girder supported like a cantilever was limited (Eq. 6)

$$w_0 = \frac{L}{250} = \frac{500\,\text{cm}}{250} = 2\,\text{cm}. \tag{6}$$

The constraints will be the area of each bar (A_0) greater than or equal to 1 cm^2. The compressive and tensile stresses (σ_0) will not exceed 21.5 kN·cm^{-2} – the task assumes the same value for compressive and tensile stresses without considering buckling. The deflection (w_0) will not exceed 2 cm. Young modulus (E) is equal to 20,500 kN·cm^{-2}. In addition, the node between bars 3–8 will be loaded with a force (P) of 100 kN.

Truss geometry

The study's subject is a truss (Fig. 1) supported by two supports. It consists of 13 bars and is loaded by a single force concentrated at the node connecting bars 3–8. The truss presented in Figure 1 is statically undetermined twice (because of two additional bars). The preconceived bar cross-sections are shown in Table 2.

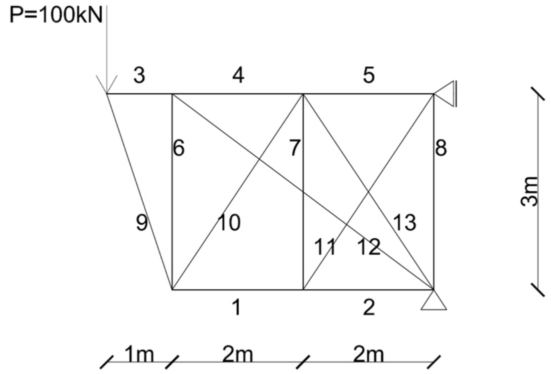


Fig. 1. Truss scheme

Source: the authors' compilation.

Table 2. Truss bars characteristics

Bars	Steel	Cross-section	Area [cm^2]
1–5	S235	RK 50 × 50 × 5	8.73
6–13	S235	RK 30 × 30 × 3	3.14

Source: the authors' compilation.

The force method

In the first step, the stresses occurring in the truss were calculated using the force method (Fig. 2) and two computer programs: MS Excel and Ansys Mechanical.

The basic layout is shown in Figure 3. Tables 3–5 show the load states from the unit forces, P forces and reactions in the supports, and the calculated forces in the bars.

Truss stresses, maximum deflection and bar volume were calculated using MS Excel. The geometry was also entered into Ansys Mechanical, and the results were obtained, which fully agree, as shown in Table 6. Stress limits were exceeded for bars S9 and S10.

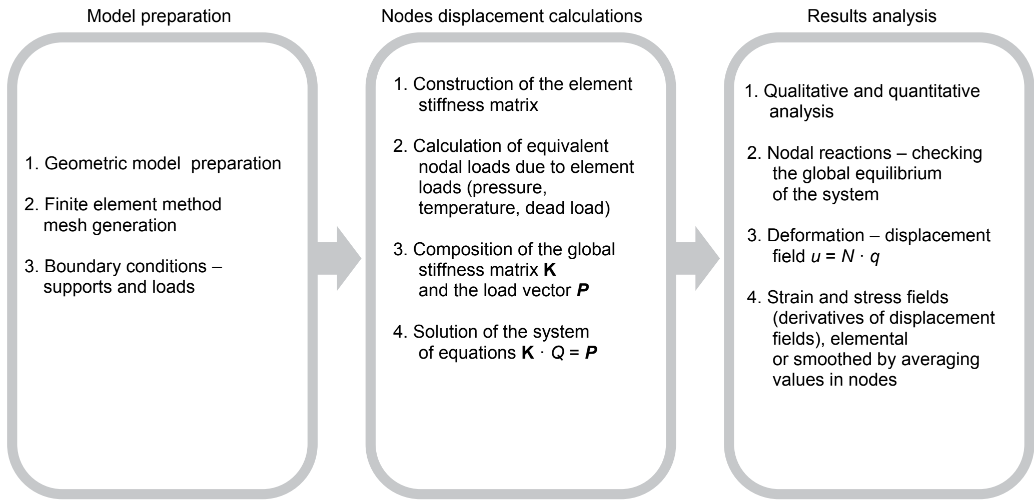


Fig. 2. General finite elements method algorithm

Source: the authors' compilation.

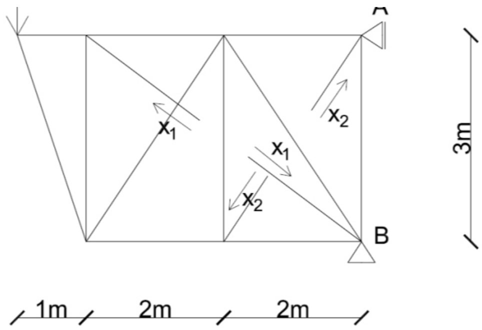
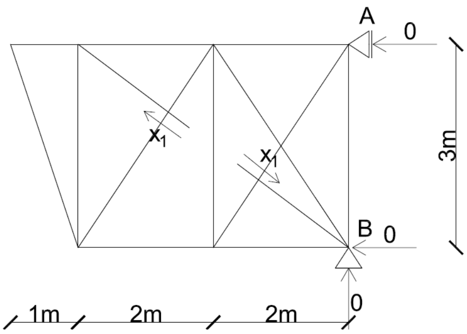
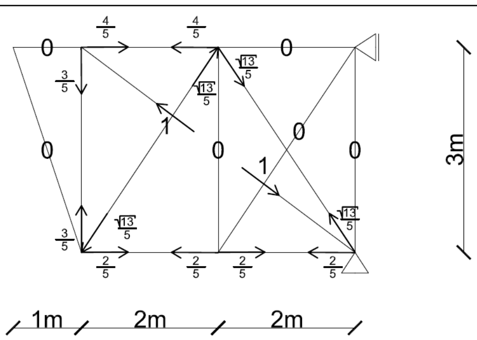


Fig. 3. Analysed truss layout

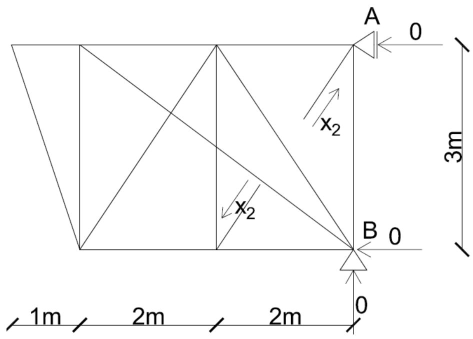
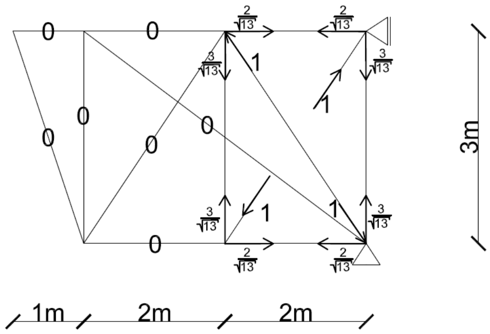
Source: the authors' compilation.

Table 3. Forces in bars for unit load $x_1 = 1$

Description	Scheme
Condition from unit load $x_1 = 1$ and reactions in supports	
Forces in the bars	

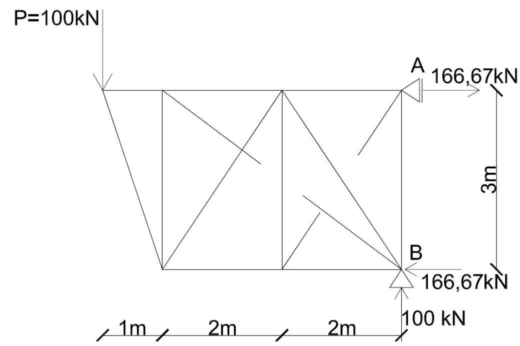
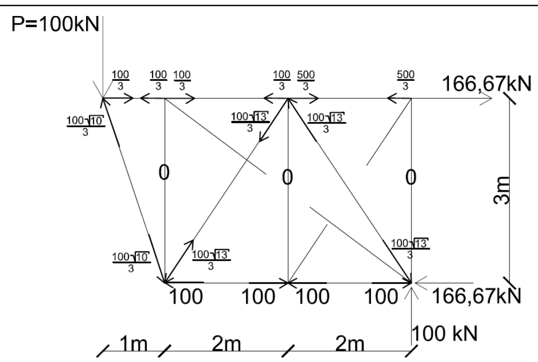
Source: the authors' compilation.

Table 4. Forces in bars for unit load $x_2 = 1$

Description	Scheme
Condition from unit load $x_2 = 1$ and reactions in supports	
Forces in the bars	

Source: the authors' compilation.

Table 5. Forces in bars for load $P = 100$ kN

Description	Scheme
Condition from load $P = 100$ kN and reactions in supports	
Forces in the bars	

Source: the authors' compilation.

Table 6. Results of a truss solved by the force method in MS Excel and Ansys Mechanical

Parameter	Unit	Ansys Mechanical	MS Excel
Bottom bars area	cm^2	8.73	8.73
Top bars area	cm^2	8.73	8.73
Diagonal bars area	cm^2	3.14	3.14
Vertical bars area	cm^2	3.14	3.14
Maximum area	cm^2	8.73	8.73
S1	$kN \cdot cm^{-2}$	–8.85515507	–8.85515507
S2	$kN \cdot cm^{-2}$	–10.5850944	–10.58509437
S3	$kN \cdot cm^{-2}$	3.81825124	3.81825124
S4	$kN \cdot cm^{-2}$	9.01744855	9.01744855
S5	$kN \cdot cm^{-2}$	17.3613169	17.36131690
S6	$kN \cdot cm^{-2}$	10.8413199	10.84131987
S7	$kN \cdot cm^{-2}$	–7.21450803	–7.21450803
S8	$kN \cdot cm^{-2}$	–7.21450803	–7.21450803
S9	$kN \cdot cm^{-2}$	–33.5698265	–33.56982654
S10	$kN \cdot cm^{-2}$	25.24584635	25.24584635
S11	$kN \cdot cm^{-2}$	8.67075954	8.67075954
S12	$kN \cdot cm^{-2}$	–18.0688665	–18.06886646
S13	$kN \cdot cm^{-2}$	–16.5750868	–16.57508680
Maximum deflection	cm	1.93736294	1.93736294
Volume	cm^3	16 642.3845	16 642.3845

Source: the authors' compilation.

Optimisation in Ansys Mechanical – the gradient method

Data were entered into the program – optimisation constraints and decision variables were defined. The program then considered the maximum allowable cross-sectional area of 100 cm^2. The decision variables will change values if the changes are greater than 0.01 cm^2. For stresses, the program will stop changing them if the changes are less than 0.01 $kN \cdot cm^{-2}$, for deflection – 0.001 cm and for volume – 0.1 cm^3.

The optimisation process to the minimum value of the function was limited to 100 steps. No value was introduced to determine how much the decision values should change at the beginning of the optimisation; the program changed them by about 5%. The program counts the derivative of the objective function for each decision variable, selects the best solution out of four, and then takes a step in that direction. The smallest value of the volume of the structure was obtained in the 28th step.

Optimisation in Ansys Mechanical – the sub-problem method

Data were entered into the program as in the gradient method. Decision variables were changed as long as no better result was obtained after 30 trials. The smallest value for the volume of the structure was obtained in the 68th step.

Optimisation in MS Excel

The bars were separated into four groups – bottom, top, diagonal, and vertical. The area fields of these bars are decision variables. The cross-sections were optimised using the solver add-on, which considered the minimum of the objective function for the truss volume. Constraints on the minimum cross-section A0, the stress limits and the maximum allowable deflection value were considered.

RESULTS

The results from MS Excel and Ansys Mechanical differ by relatively minimal values, as shown in Table 7. MS Excel performed better optimisation (lower steel volume by 0.19%), but Ansys Mechanical calculated 0.18% lower deflection.

Table 7. Truss optimisation results were obtained with Ansys Mechanical (the gradient and sub-problem method) and in MS Excel

Parameter	Unit	Ansys Mechanical		MS Excel with Solver
		Gradient method	Sub--problem method	
Bottom bars area	cm^2	4.2197	4.0065	4.000977
Top bars area	cm^2	7.5473	7.4003	7.386617
Diagonal bars area	cm^2	1.5951	1.5283	1.52326
Vertical bars area	cm^2	4.9169	4.9117	4.902756

Table 7 (cont.)

Parameter	Unit	Ansys Mechanical		MS Excel with Solver
		Gradient method	Sub--problem method	
Maximum area	cm^2	7.5473	7.4003	7.386617
S1	$kN \cdot cm^{-2}$	–18.521	–19.508	–19.53687897
S2	$kN \cdot cm^{-2}$	–20.454	–21.469	–21.49999958
S3	$kN \cdot cm^{-2}$	4.4166	4.5043	4.51266571
S4	$kN \cdot cm^{-2}$	10.206	10.408	10.42427342
S5	$kN \cdot cm^{-2}$	21.002	21.460	21.49999999
S6	$kN \cdot cm^{-2}$	20.545	21.439	21.49999990
S7	$kN \cdot cm^{-2}$	–7.6702	–7.7133	–7.73446598
S8	$kN \cdot cm^{-2}$	–7.6702	–7.7133	–7.73446598
S9	$kN \cdot cm^{-2}$	–21.438	–21.461	–21.50000000
S10	$kN \cdot cm^{-2}$	16.433	16.452	16.48549022
S11	$kN \cdot cm^{-2}$	2.9906	2.8844	2.88811488
S12	$kN \cdot cm^{-2}$	–11.109	–11.1188	–11.13322320
S13	$kN \cdot cm^{-2}$	–13.442	–13.567	–13.59737534
Maximum deflection	cm	1.7422	1.7734	1.77668250
Volume	cm^3	1 6229	16 000	15 969.5404

Source: the authors' compilation.

For the analysed truss, the sub-problem method proved more effective than the gradient method. The result obtained has enough accuracy to compare the results with those of MS Excel – the volume differs by 0.19%. MS Excel was better at optimisation, as it achieved a lower volume of steel. In contrast, the lower deflection was achieved by Ansys Mechanical (by 0.18%). Stress proved to be the decisive optimisation parameter in this case. The optimisation saved about 4% of the steel volume.

DISCUSSION AND CONCLUSIONS

The method used influences the result of the optimisation. The presented approach helps to understand the structure's operation better and provides higher efficiency. Very similar results were obtained, and each optimised the final result. Effective structural optimisation reduces the cost of a construction project due to less material used. It also translates into lower emissions of pollutants into the atmosphere.

There are many opportunities for further research on the topic, as the issue of optimisation is very complex. Another analysed issue could occur with a different division of bars into groups – instead of dividing the bars into four different groups, they could all be treated individually. They could also be divided in other ways, for example, considering the stresses that occur in them.

The studies are helpful for designers, structural engineers, architects and anyone involved in optimisation in the construction industry. Both practitioners and researchers can benefit from them.

Authors' contributions

Conceptualisation: M.K. and T.S.; methodology: M.K. and T.S.; validation: T.S.; formal analysis: M.K. and T.S.; investigation: M.K.; resources: M.K., A.Ch. and T.S.; data curation: M.K.; writing – original draft preparation: M.K.; writing – review and editing: T.S.; visualisation: M.K.; supervision: T.S.; project administration: M.K.; funding acquisition: M.K. and A.Ch.

All authors have read and agreed to the published version of the manuscript.

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OPTYMALIZACJA WIELKOŚCIOWA KRATOWNICY STALOWEJ WEDŁUG ALGORYTMÓW

STRESZCZENIE

Metody obliczeniowe umożliwiają dyskretyzację matematyczną w projektowaniu konstrukcji. W ten sposób uzyskana macierz wyników, dzięki projektowaniu algorytmicznemu, prezentuje różne efektywne konstrukcyjnie elementy nośne. Głównym celem artykułu jest podjęcie tematu optymalizacji materiałowej w kratownicowych konstrukcjach prętowych przy zastosowaniu różnych podejść. W artykule przeanalizowano i porównano optymalizację kratownicy stalowej z zastosowaniem dwóch programowych algorytmów optymalizacyjnych (MS Excel i Ansys Mechanical) według metod gradientowych i programowania dynamicznego (rozwiązań podproblemów). Kluczowe wnioski przedstawiają skuteczność każdej z metod w optymalizacji strukturalnej i kończą niniejsze opracowanie instrukcją do efektywnego projektowania konstrukcyjnego elementów kratownicy nośnej.

Słowa kluczowe: optymalizacja konstrukcji, metody komputerowe w inżynierii lądowej, optymalizacja kratownicy, metoda elementów skończonych