Acta Sci. Pol.
Architectura 22 (2023), 30–37

aspa.sggw.edu.pl ISSN 1644-0633 (suspended) eISSN 2544-1760 DOI: 10.22630/ASPA.2023.22.4

ORIGINAL PAPER

Received: 21.02.2023
Accepted: 14.04.2023

EVALUATION OF THE INFLUENCE OF LINEAR STRESS CONCENTRATORS IN REINFORCED CONCRETE ELEMENTS USING THE POSTULATES OF FRACTURE MECHANICS

Dzmitry Salauyou[1]✉, Piotr Knyziak[2]

[1]Institute of Civil Engineering, Warsaw University of Life Sciences – SGGW, Warsaw, Poland
[2]Faculty of Civil Engineering, Warsaw University of Technology, Warsaw, Poland

ABSTRACT

The article is devoted to the study of linear stress concentrators in reinforced concrete, which, unlike randomly located stress concentrators from a large and medium-sized concrete aggregate (granite or gravel crushed stone), can have a strategic impact on the formation of cracks in a reinforced concrete element in the presence of a sufficient level of tensile stresses. The study is one of the theoretical parts of a significant research program on the cracking of reinforced concrete elements in tension and bending. The description of the process of cracking with the help of fracture mechanics allows us to try to formulate a mathematical apparatus for taking into account stress concentrators linearly located in reinforced concrete elements and their influence on the formation of cracks, in particular, at the moment of cracking in bending elements. Field tests of specimens with artificially created stress concentrators were carried out, including the RILEM method, on the basis of which trial calculations of the cracking moment were carried out using the stress intensity factor.

Keywords: fracture mechanics, the stress intensity factor, cracking moment

INTRODUCTION

The concentration of stresses in concrete, according to the studies of Bažant and other researchers according to the basics of fracture mechanics, occurs in places where there are pores, cavities, and areas with increased or decreased strength. In the works of Bressan, Effting and Tramontin (1991) and the work of Golewski and Sadowski (2007, 2010), it is indicated that in a reinforced concrete element, micro-cracks appear at the contact edge of the aggregate and the concrete mass during stretching (Fig. 1). However, the aggregate is chaotic in the concrete and therefore, at certain stages of loading, micro-cracks do not develop into main cracks, because the mass of concrete prevents the development of these micro-cracks, as stated in the papers by Hillerborg, Modéer and Petersson (1976), the American Concrete Institute (ACI Committee 446, 1992; ACI Committee 224, 1997; Committee 224, 2001), Bažant (1992), Prokopski (2009), and Bažant and Cedolin (2010).

As confirmed by the research conducted earlier by Salauyou (2009), Knyziak and Salauyou (2010), Salauyou and Knyziak (2010a, 2010b), the spacing of cross-section bars of horizontal meshes and skeletons affects the spacing of cracks in reinforced concrete elements in tension and bending. This phenomenon has also been described in studies by Lee, Mansur, Tan and Kasiraju (1964, 1987), and Nawy (1964). The mechanism of crack formation at the location of the cross-section bars can be described by means of fracture mechanics.

The cross-section bar placed in the concrete differs from the aggregate in that it occupies a specific,

✉dzmitry_salauyou@sggw.edu.pl

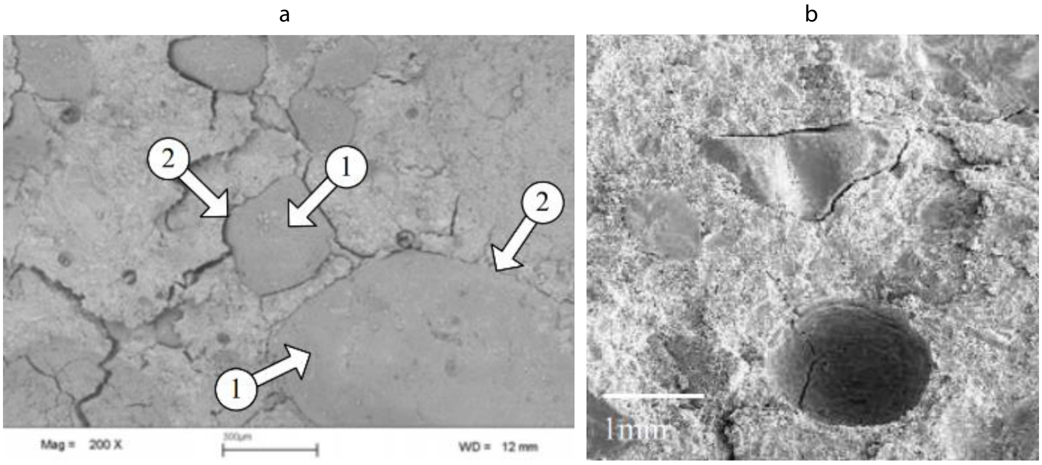

Fig. 1. Micro-cracks appear at the contact edge of the aggregate and the concrete mass: 1 – aggregate grains, 2 – micro--cracks in contact layers

Source: [a]Golewski and Sadowski (2007); [b]Bressan, Effting and Tramontin (1991).

linearly oriented position in the concrete (a position perpendicular to the bending moment while bending elements). When tensile stress appears and increases in concrete, micro-cracks appear on the edge of the cross-section bar and concrete mass, as well as on the edge of aggregate and concrete mass. With a further increase in tensile stresses in the concrete, micro-cracks develop along the entire boundary of the bar with the concrete, and the cross-section bar loses its cohesion with the concrete. This means that in the concrete at the location of the cross-section bar, there is a weakening equal to its diameter (Fig. 2).

If the cross-section is weakened with a circular or elliptical hole, then, according to the Inglis equation (1913), the stress concentration factor $K = 1 + 2a / b$ (a and b are radii of the horizontal and vertical axes, respectively) is applied to the elliptical cross-section in tension. Accordingly, for a circle, the $K = 3$. In fur-

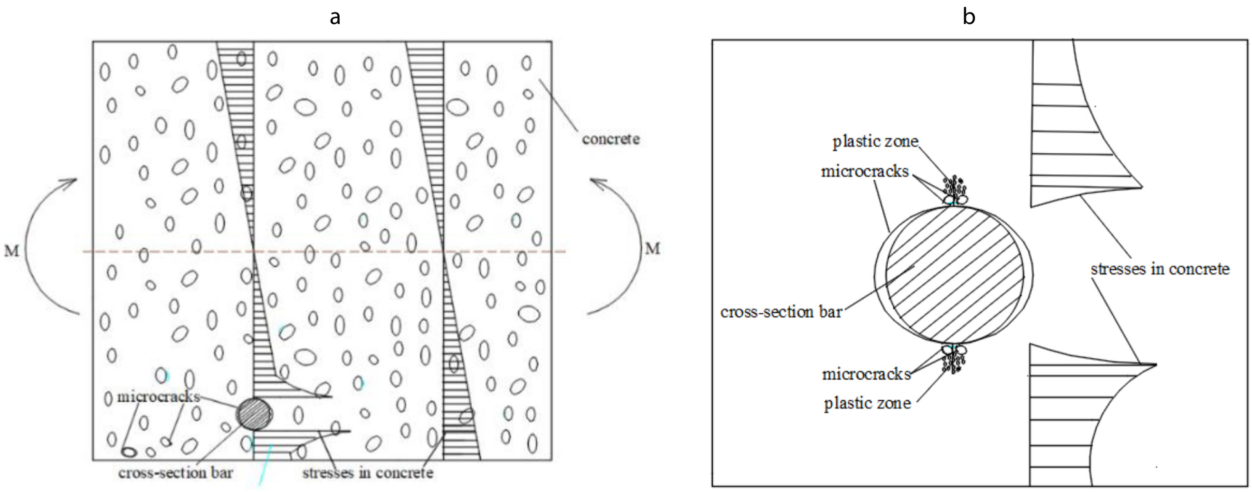

Fig. 2. Tensile stresses in concrete. The appearance of the initial crack in the fracture process zone near the linear stress concentrator transverse reinforcing bar

Source: [a]Salauyou (2009); [b]Salauyou and Knyziak (2010).

ther research, the researchers found that a $K = 3$ is correct for a very wide plane and that stress concentration factors are determined by empirical testing for narrow planes. Table 1 presents the values of the stress concentration factor depending on the ratio of the opening width to the width of the plane.

Table 1. The value of the stress concentration factor (K) depending on the scale of the hole to the width of the plane

The size of the scale of the hole to the width of plane	0	0.1	0.2	0.3	0.4	0.5
The magnitude of the stress concentration factor	3.00	3.03	3.14	3.36	3.74	4.32

Source: calculated with using Kirsch equations.

This means that the stress concentration in the concrete next to the cross-section bar is more than three times that of the stress in the cross-section of the concrete without weakening and that an initial crack will necessarily be formed at the location of the cross-section bar (Fig. 2).

For concrete, as for a non-homogeneous material, special coefficients can be applied that describe the stress concentration at the crack tip. For a crack of normal separation, which are cracks normal to the longitudinal axis of the element from the action of a bending moment, such a coefficient is K^s_{Ic}, determined empirically by testing samples, according to the RILEM methodology.

The equation is used for the determination of K^s_{Ic} in the work of Karihaloo (1995):

$$K^s_{Ic} = 6Y(\alpha)\, M_{cr}\, a^{0.5} / b\, d^2,$$

$$M_{cr} = K^s_{Ic}\, b\, d^2 / 6Y(\alpha)\, a^{0.5},$$

where:
$Y(\alpha)$ – geometric function.

For a beam or slab freely supported with one load point:

$$Y(\alpha) = [1.99 - \alpha(1 - \alpha)\,(2.15 - 3.93\alpha + 2.7\alpha^2] / [(1 + 2\alpha)\,(1 - \alpha)^{3/2}].$$

For a beam or slab freely supported with two load points:

$$Y(\alpha) = 1.99 - 2.47\alpha + 12.97\alpha^2 - 23.17\alpha^3 + 24.8\alpha^4 + 60.5\alpha^{16},$$

$$\alpha = a / d,$$

$$M = M_1 + M_2,$$

where:
M_1 – bending moment from the applied load,
M_2 – bending moment of self-weight action,
a – initial notch (in the RILEM method), equal to the length of the initial crack.

In this way, knowing K^s_{Ic} (if there is a possibility to determine it through material tests for a given concrete), we can approximately calculate M_{cr}.

EXPERIMENTAL ELEMENTS AND TESTING PROCEDURE

Nine reinforced concrete beams with dimensions of 1,200 × 250 × 150 mm were used with the reinforcement of 12 mm diameter S500 bars. All of them are from concrete classes 25/30. A description of experimental elements (experimental beams), with dimensions of 1,200 mm long, 250 mm thick and 150 mm wide, is presented in Table 2. The design of the beams differed: three elements with a cross-section bar, and three elements with a through hole instead of a cross--section bar, and three control elements (without any cross-sectional weakness). The central zone of the beams is shown in Figure 3.

Table 2. Experimental elements/beams

No	Pattern code	Peculiarities of the pattern
1	B 1-1	with a cross-section bar
2	B 1-2	with a cross-section bar
3	B 1-3	with a cross-section bar
4	B 2-1	with a hole instead of a cross-section bar
5	B 2-2	with a hole instead of a cross-section bar
6	B 2-3	with a hole instead of a cross-section bar
7	B 3-1	without any cross-sectional weakness
8	B 3-2	without any cross-sectional weakness
9	B 3-3	without any cross-sectional weakness

Source: own work.

a b

Fig. 3. Beams with a hole instead of a cross-section bar (a) and beams with a cross-section bar (b)

Source: own work.

All the experimental elements/beams were tested in the same way for bending on a special test stand with the help of a hydraulic cylinder. Figure 4 shows the research scheme and the view of the test stand. During the experiment, the deflection of the beams and the width of the cracks were measured. The width of the cracks was determined using a clock-type indicator (with a measuring accuracy of 0.001 mm) set on special holders and an optical microscope. The deflection of the beams was measured using a clock-type indicator with an accuracy of 0.01 mm, set on a special frame.

Also, four concrete beams (concrete classes 25/30) with dimensions of 1200 × 250 × 150 mm with a special notch were made and tested according to the RILEM method in order to determine the coefficient K_{Ic} that characterises the normal detachment crack according to the fracture mechanics approach. Figure 5 shows the beams for testing according to the RILEM method, during and after testing.

The beams were tested in the same way for bending on a special test bench using hydraulic cylinders according to the RILEM method (three load – unload cycles). Figure 6 shows the research scheme and the

a b

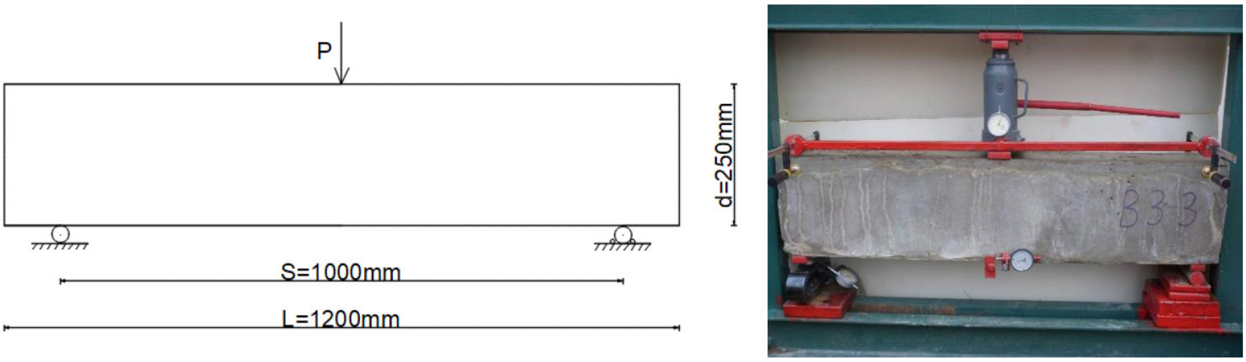

Fig. 4. Scheme of conducting tests, including for determining the coefficient K_{Ic}, according to the RILEM method (a); view of the test stand (b)

Source: own work.

a b

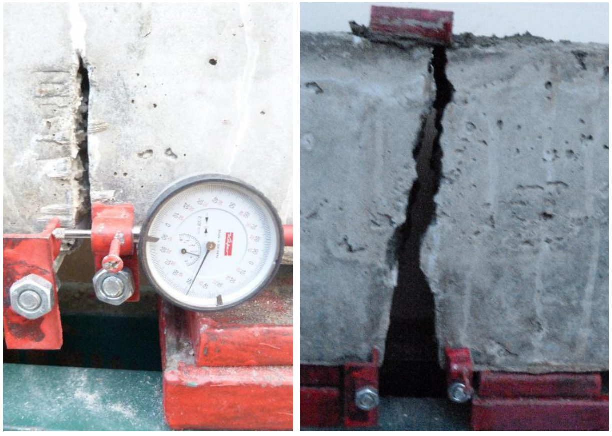

Fig. 5. Beams for testing, according to the RILEM method, the during the test (a) and after testing (b)

Source: own work.

a b

Fig. 6. Scheme of testing of four experimental elements/beams, according to the RILEM method (a); view of the test stand (b)

Source: own work.

view of the test stand. During the experiment, the width of the cracks was determined using a clock-type indicator (with a measuring accuracy of 0.001 mm) set on special holders.

RESULTS AND DISCUSSION

As a result of testing beam samples according to the RILEM method, it was possible to obtain the relevant data necessary to determine the stress intensity factor (K_{Ic}). A characteristic diagram of the dependence of deformations from the load measured in the centre of the beam directly in the zone of artificial damage to the section using a dial indicator with a division value of 0.001 mm is shown in Figure 7.

Young's modulus (E) is calculated from the equation (Shah & Carpinteri, 1991):

$$E = 6S\, a_0\, V_1(\alpha) \,/\, (C_i\, d^2\, b);\ \alpha = a_0 \,/\, d = 1/3;$$
$$C_i = 2.8\ 10^{-9}\ \mathrm{m{\cdot}N^{-1}};\ d = 0.25\ \mathrm{m};\ b = 0.15\ \mathrm{m},$$

$$V_1(\alpha) = 0.76 - 2.28\alpha + 3.87\alpha^2 - 2.04\alpha^3 + 0.66 \,/\, (1 - \alpha)^2 =$$
$$= 1.83944;\ E = 34.896.8\ \mathrm{mPa}.$$

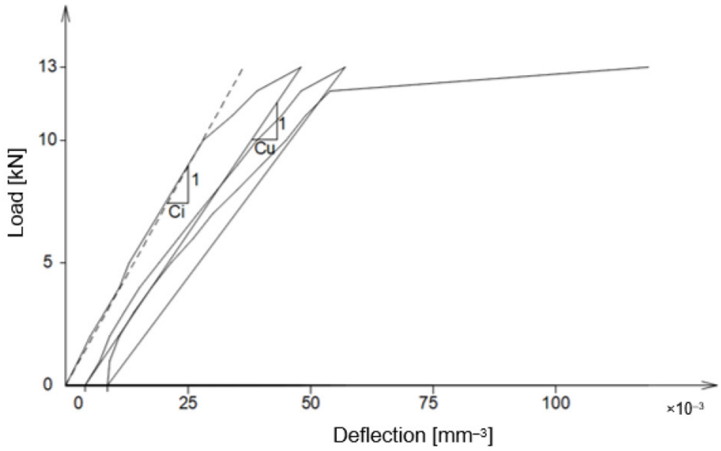

Fig. 7. Dependence of deformations from the load according to the RILEM method

Source: own work.

The critical effective crack length $a_c = a_0$ + stable crack growth at peak load:

$a_c = E\ C_u\ d^2\ b\ /\ [6S\ V_1(\alpha)]$; $C_u = 3.38461 \cdot 10^{-9}$ m·N^{-1}; $a_c = 0.100329$ m.

The critical stress intensity factor $K^s_{Ic} = 3(P_{max} + 0.5W)\ S(\pi\ a_c)^{0.5}\ F(\alpha_1)\ /\ (2\ d^2\ b)$:

$P_{max} = 13{,}000$ N, $W = W_0 S / L$, W_0 – self-weight of the beam = 1,102.5 N, $\alpha_1 = a_c / d = 0.401316$,

$$F(\alpha_1) = 1.99 - \alpha_1(1 - \alpha_1)\ (2.15 - 3.93\alpha_1 + 2.7\alpha_1)\ /\ [\pi^{0.5}(1 + 2\alpha_1)\ (1 - \alpha_1)]^{1.5} = 1.43673,$$

$K^s_{Ic} = 1.735159$ mPa·m$^{0.5}$.

The analysis of the test results of 9 experimental element/beams showed that the failure of all elements occurred at the same value of the bending moment (13 kNm). The magnitude of the cracking moment for all three elements with a cross-section bar was three elements with through hole 7 kNm. The value of the cracking moment for all s was also 7 kNm. The value of the cracking moment for all three control elements (without a cross-section bar and without a hole) was 10 kNm. Figure 8 shows the elements during the tests, after cracking.

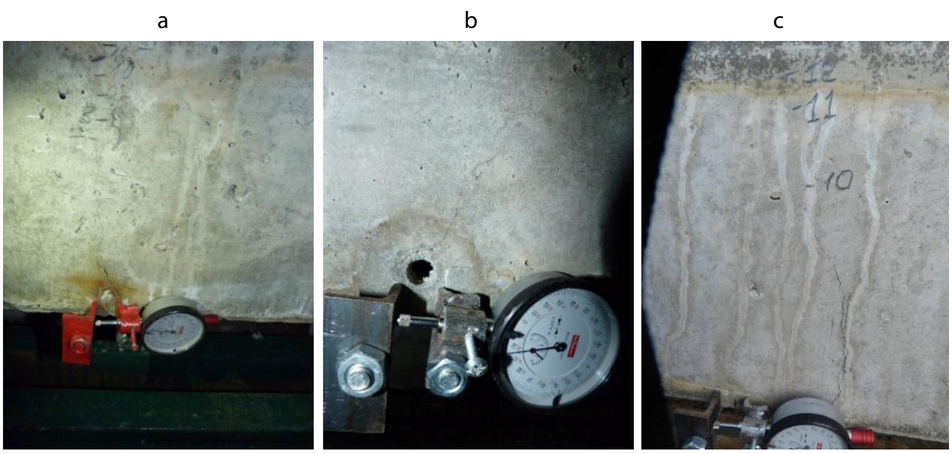

Fig. 8. Cracking of beams – with a cross-section bar (a), with a through hole instead of a cross-section bar (b), without a cross-section bar and without a hole (c)

Source: own work.

We determine the moment of cracking for beams with a transverse rod and beams with a through hole analytically according to the formulas indicated in the Karihaloo (1995) for a notched beam with free support. For the height of the notch, we take the upper point of the transverse rod or the upper point of the through hole (we accept this assumption, taking into account the fact that the thickness of the concrete cover between a through hole or rebar and the surface of a reinforced concrete element is 15 mm, which is less than the maximum size of large concrete aggregate and less than stable crack growth at peak load):

$a = 27$ mm, $\alpha = a / d = 27$ mm / 250 mm = 0.108,

$$Y(\alpha) = [1.99 - \alpha(1 - \alpha)\ (2.15 - 3.93\alpha + 2.7\alpha^2)] / [(1 + 2\alpha)\ (1 - \alpha)^{3/2}],$$

$$Y(\alpha) = [1.99 - 0.108(1 - 0.108)\ (2.15 - 3.93 \cdot 0.108 + 2.7 \times 0.108^2)] / [(1 - 2 \cdot 0.108)\ (1 - 0.108)^{3/2}] = 1.7773186,$$

$$M_{cr} = (K^s_{Ic}\ b\ d^2) / (6Y(\alpha)\ \mathrm{a}^{0.5}) = (1.735159\ \mathrm{mPa{\cdot}m^{0.5}} \times 0.15 \cdot 0.25^2) / (6 \cdot 1.7773186 \cdot 0.027^{0.5}),$$

$$M_{cr} = 9{,}283.5\ \mathrm{kNm};\ M_{cr} = (M_1 + M_2),$$

$$M_1 = M_c - M_2;\ M_2 = (W / 2)\ (S / 2);\ W = W_0 S / L,$$

where:

M – bending moment of external load,

M_1 – bending moment from applied load, M_1 = 9,283.5 kNm – (1,102.5 · 1/2 1.2) (1/2) = 9,053.8 kNm,

W_0 – self-weight of the beam, W_0 = 1,102.5 N.

The analytically calculated moment of cracking is somewhat different from the actual value of the moment from the applied load at which a crack was formed in samples with a transverse reinforcing bar and in samples with a through hole (7 kNm); therefore, the mathematical apparatus needs to be improved and refined. But in view of the fact that the calculated moment we received is less than the moment of cracking of control samples (without damage to the section) – 10 kNm, this confirms the correctness of the general direction of theoretical research to determine the moment of cracking of the section of a reinforced concrete element with transverse reinforcing bars with the help of fracture mechanics postulates.

CONCLUSIONS

As a result of the studies carried out, it was possible to obtain the relevant values of the stress intensity factor (K_{Ic}) according to the RILEM methodology. These data will be used in the future to develop and test a mathematical apparatus for the analytical calculation of the bending moment of cracking for reinforced concrete elements with linear section damage located perpendicular to the longitudinal axis, including at the location of transverse reinforcement bars.

Experimental studies confirmed the theories about the inevitability of cracks at the location of the cross--section bars and about the impact of the presence of the cross-section bars on the magnitude of the cracking moment.

As a result of bending tests of nine reinforced concrete beams, it was confirmed that cracks always appeared in the place of placing the cross-section bars or in the place of placing the through hole, and with the same value of the cracking moment (30% lower than in the control elements), which confirms the theories that the presence of a cross-section bar causes stress concentrations in the concrete, as well as the presence of a through hole of the same diameter.

Authors' contributions

Conceptualisation: D.S. and P.K.; methodology: D.S.; formal analysis: D.S. and P.K.; investigation: D.S.; recourses D.S.; data curation D.S.; writing – original draft preparation: D.S. and P.K.; writing – review and editing: D.S. and P.K.; visualization: D.S.

All authors have read and agreed to the published version of the manuscript.

REFERENCES

ACI Committee 446 (1992). *Fracture Mechanics of Concrete Structures: Concepts, Models and Determination of Material Properties* (ACI 446.1R-91). American Concrete Institute.

ACI Committee 224 (1997). *ACI Committee, Cracking of Concrete Members in Direct Tension* (ACI 224.2R-92). American Concrete Institute.

ACI Committee 224 (2001). *Control of Cracking in Concrete Structures* (ACI 224R-01). American Concrete Institute.

Bažant, Z. P. (Ed.) (1992). *Fracture Mechanics of Concrete Structures*. Proceedings of the First International Conference on Fracture Mechanics of Concrete Structures (FraMCoS1). Boca Raton: CRC Press.

Bažant, Z. P. & Cedolin, L. (2010). *Stability of Structures. Elastic, Inelastic, Fracture and Damage Theory*. World Scientific Publishing.

Bressan, J. D., Effting, C. & Tramontin, A. (1991). *Fracture Toughness and Strength of Fly Ash Concrete*. Centro de Ciencias Tecnologicas – Universidade do Estado de Santa Catarina (UDESC).

Golewski, G. & Sadowski T. (2007). Analiza kruchych uszkodzeń w kompozytach betonowych. *Czasopismo Techniczne. Budownictwo*, *104* (1-B), 55–62.

Golewski, G. L. & Sadowski T. (2010). Model betonu z uszkodzeniem w ujęciu mezomechanicznym. *Modelowanie Inżynierskie*, *39*, 69–79.

Hillerborg, A., Modéer, M. & Petersson, P-E. (1976). Analysis of Crack Formation and Crack Growth in Concrete by Means of Fracture Mechanics and Finite Elements. *Cement and Concrete Research*, *6* (6), 773–782.

Inglis, C. E. (1913). Stresses in Plates Due to the Presence of Cracks and Sharp Corners. *Transactions of the Institute of Naval Architects*, *55*, 219–241.

Karihaloo, B. L. (1995). Fracture mechanics and structural concrete. *International Journal of Fracture*, *77*, R19.

Knyziak, P. & Salauyou, D. (2010). Wpływ rozstawu prętów poprzecznych w siatkach spawanych na morfologię rys [Influence of a spacing of cross-section reinforcing bars of welded grids on cracks morphology in reinforced concrete elements]. In *Theoretical Foundations of Civil Engineering. Polish-Ukrainian-Lithuanian Transactions* (Vol. 18, pp. 353–360). Warszawa: Oficyna Wydawnicza Politechniki Warszawskiej.

Lee, S. L., Mansur, M. A., Tan, K. H. & Kasiraju, K. (1964). Cracking Behavior of One-Way Slabs Reinforced with Welded Wire Fabric. *ACI Structural Journal*, *March–April*, 208–216.

Lee, S. L., Mansur, M. A., Tan, K. H. & Kasiraju, K. (1987). Cracking Behavior of Concrete Tension Members Reinforced with Welded Wire Fabric. *ACI Structural Journal*, *November–December*, 481–491.

Nawy, E. G. (1964). Flexural Cracking in Two-Way Concrete Slabs Reinforced with High Strength Welded Wire Fabric. *Journal of the American Concrete Institute*, *August*, 997–1007.

Prokopski, G. (2009). *Mechanika pękania betonów cementowych*. Rzeszów: Oficyna Wydawnicza Politechniki Rzeszowskiej.

Salauyou, D. (2009). *Research of Influence of a Step of Cross-Section Reinforcing Bars of Welded Grids on Cracking of Reinforced Concrete Elements*. Paper presented at the II International Interdisciplinary Technical Conference of Young Scientists 20–22 May 2009, Poznań, Poland.

Salauyou, D. & Knyziak, P. (2010a). *Crack Modeling in Reinforced Concrete Elements with Different Step of Cross-Section Reinforcing Bars*. Paper presented at the III International Interdisciplinary Technical Conference of Young Scientists 19–21 May 2010, Poznań, Poland.

Salauyou, D. & Knyziak, P. (2010b). *Research and Modeling of Influence of a Step of Cross-Section Reinforcing Bars of Welded Grids on Cracking Reinforced Concrete Elements*. Paper presented at the XIX Slovak-Polish-Russian Seminar "Theoretical Foundations of Civil Engineering". Žilina, Slovakia, September 2010.

Shah, S. P. & Carpinteri, A. (1991). *Fracture Mechanics. Test Methods for Concrete* (Technical Committee 089-FMT. RILEM report 5). London–New York–Tokyo–Melbourne–Madras: Chapman and Hall.

OCENA WPŁYWU LINIOWYCH KONCENTRATORÓW NAPRĘŻEŃ W ELEMENTACH ŻELBETOWYCH Z WYKORZYSTANIEM POSTULATÓW MECHANIKI PĘKANIA

STRESZCZENIE

Artykuł poświęcono badaniu liniowych koncentratorów naprężeń w żelbecie, które w przeciwieństwie do losowo rozmieszczonych koncentratorów naprężeń z dużego i średniego kruszywa betonowego (tłuczeń granitowy lub żwirowy) mogą mieć istotny wpływ na powstawanie spękań w elemencie żelbetowym w przypadku wystarczającego poziomu naprężeń rozciągających. Praca jest jedną z teoretycznych części znaczącego programu badawczego dotyczącego pękania przy rozciąganiu i zginaniu elementów żelbetowych. Opis procesu pękania za pomocą mechaniki pękania pozwala na próbę sformułowania aparatu matematycznego uwzględniającego koncentratory naprężeń liniowo rozmieszczone w elementach żelbetowych i ich wpływ na powstawanie rys, w szczególności w momencie pękania elementów zginanych. Przeprowadzono badania terenowe próbek ze sztucznie wytworzonymi koncentratorami naprężeń, w tym metodą RILEM, na podstawie których przeprowadzono próbne obliczenia momentu pękającego z wykorzystaniem współczynnika intensywności naprężeń.

Słowa kluczowe: mechanika pękania, współczynnik intensywności naprężeń, moment rysujący